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Controlling dynamics in spatially extended systems

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Spatially extended systems exhibit a variety of spatiotemporal dynamics—from stable to chaotic. These dynamics can change under pathological conditions and impair normal functions. Thus, having the ability to control the altered dynamics for improved functioning has the potential for wide ranging applications in real and artificial systems. Here we propose a simple and general method that can be used to target the spatiotemporal dynamics, both globally and in spatially localized regions, in either direction—i.e., towards the stable or unstable manifold—by simply changing the strength and the sign of an externally applied *perturbation* or *pinning*. The method is applicable to both chaotic and nonchaotic systems, with discrete and continuous local dynamics, and for different topologies of interactions. We also apply it to simulate an experiment on epileptogenic neuronal activity in rat hippocampal tissue [B. J. Gluckman *et al.*, J. Neurophys. **76**, 6202 (1996)]. This unified approach for differential targeting of global and local dynamics promises to be useful for systems spanning large spatial scales and having structural and functional heterogeneity.

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I. INTRODUCTION

Along with homeostasis, periodic, complex, and even chaotic spatiotemporal dynamics are shown to be abundantly present in a variety of real and artificial systems, such as in arrays of semiconductors, lasers, plasmas, and chemical reactors, fluid flow, and in cardiac and neural tissues [2–8]. Many systemic or environmental factors can change the normal dynamics and abnormalities or disease can set in. Examples are instabilities in lasers, charge density waves in plasmas, and arrays of Josephson junctions [9]; desynchronization in coupled chemical reactors; defective biochemical functions; cardiac arrhythmia, epileptogenic neural activity, and pathological physiology [4,10]; and, large population fluctuations and epidemics in metapopulation [11,12]. Engineering complex dynamics is also becoming increasingly useful for improved functioning—for information transmission in communication sciences [13], mixing flows in physics of fluids [14], and, in many branches of biological sciences with medical applications, such as, in the treatment of cardiac and neural diseases [15]. Thus possessing the ability to modify or have control over the dynamics of spatiotemporal systems have important applications.

It is clear from the above that there are two aspects of ‘‘control’’—it can indicate, on one hand, suppressing chaos in the dynamics and restoring the system to its regular behavior; or involve inducing/maintaining complexity in the dynamics depending on the desired performance of the system. Several ingenious theoretical approaches have been proposed for control [16], and a few have also been applied to experimental systems [4–6,17,18]. However, these methods either require *a priori* knowledge of the system dynamics, such as, the stable or unstable fixed points and periodic orbits; or, involve direct modification or tracking of the system parameters, and monitoring of time-series data for calculating representative dynamical quantities (e.g., Lyapunov exponents). This requires fairly intricate, sensitive and sophisticated experimental methodology, thereby making it difficult for implementation. This is particularly true when the systems are heterogeneous and large such as, ecological systems [19].

Any approach that is not system dependent and which can also be applied locally or globally, would offer a major advancement in real world applications. With this aim we propose here a simple and general approach that can modify the spatiotemporal dynamics of a system based on the external application of a constant perturbation or *pinning signal* in the spatial domain. Here, the sign and the strength of the pinning signal alone determine the control of the spatiotemporal dynamics to desired target states, and no prior information of the system parameters or their modification is required. In addition to effecting global control, this method is also applicable for controlling dynamics in spatially localized regions leaving the rest of the system undisturbed. Furthermore we show that, simply by pinning a small region at random, one can confer global control of dynamics in a spatially extended system having few nonlocal connections. In a preliminary study [20] we had described the efficacy of the pinning approach for suppressing chaos in a specific model system. We now show the generality of the approach for both suppressing and inducing/maintaining spatiotemporal chaos for a variety of discrete and continuous local nonlinear processes: (a) a single discrete equation (the Logistic map), (b) coupled discrete equations (the Host-parasitoid system), and (c) coupled differential equations (the Lorenz system). We, then, use a reverse approach and simulate an experiment in neurophysiology [1] to show that this simple method can successfully reproduce the experimental data.

The importance and usefulness of this approach stems from the fact that it can target the system dynamics (both locally and globally) in either direction—stable or unstable—by simply changing the strength and sign of pinning. This is of importance in systems where evolution of spatiotemporal heterogeneity requires differential targeting to

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retain normal functionality (e.g., in ecology or physiology). Also, synchronization of dynamics across space and time in coupled systems has shown to confer both a positive role in maintaining functions [21] and a negative role in increasing risk of extinction [12]. Thus any approach that can differentially control dynamics in space and time will be useful.

II. THE METHOD

Many spatially extended systems are composed of discrete entities governed by their local dynamics, which are coupled to each other through diffusion, convection, conduction, etc., [8,22,23]. Thus the spatiotemporal system considered here is given by the one-dimensional diffusively coupled lattice model with periodic boundary conditions

$$x_k(i,n+1) \equiv F_k(x_k(i,n)) = (1-\epsilon) f_k(x_m(i,n)) + \epsilon/2[f_k(x_m(i-1,n)) + f_k(x_m(i+1,n))], \tag{1}$$

where f defines the local nonlinear dynamics (described by m variables and k coupled equations) on the discrete lattice sites $i=1,2,\ldots,L$. The continuous state variable $x(i,n)$ is evaluated in discrete time steps $n=1,2,\ldots,N$, and, ϵ is the diffusive coupling strength of the nearest neighbors.

The proposed method for controlling the spatiotemporal dynamics involves applying a constant perturbation or *pinning* to the state space variable on the lattice sites in the following manner:

$$x(i,n+1) = F(x(i,n)) + p(i,n), \tag{2}$$

where $p(i,n)$ represents the strength of pinning on the ith site at nth time step. We have considered two types of pinning for achieving the desired target dynamics: (a) *uniform pinning*—perturbation of the same strength applied to all the sites at all time steps, i.e., $p(i,n)=p$, and (b) *nonuniform pinning*—pinning applied to the sites in a spatially inhomogeneous manner. Here we define $p(i,n)=\delta(i-mi_p)p$, for $m=1,2,\ldots,L/i_p$, such that, if $\delta(i-mi_p)=1$, then every site that is a multiple of i_p is "pinned" and takes a finite value p, else $p(i,n)=0$. Thus when $i_p=2$, every alternate site of the lattice is pinned. Control of the dynamics can be achieved by varying the strength and sign of $p(i,n)$, which depend on the local functional form f [24], the initial dynamical state, and the desired target state of the system. In the following sections, we first show the results of implementing this simple, unified approach in detail for the coupled map lattice (CML) with the logistic map as the local nonlinear function and then describe its application to higher-dimensional systems.

III. RESULTS AND DISCUSSION

A. Case I: Controlling the logistic CML

The logistic map given by $f(x)=rx(1-x)$, for $1\leqslant r\leqslant 4$ and $0\leqslant x\leqslant 1$, exhibits dynamics progressing from equilibrium to chaos through a sequence of period-doubling bifurcations with increasing values of r. The constant pinning p is applied to the map in the following form:

$$x(n+1) = rx(n)[x(n)-1] \pm p.$$

On scaling out p, the above equation can be written as

$$x(n+1) = Rx(n)[x(n)-1],$$

where

$$(R-1)^2 = (r-1)^2 \pm 4rp. \tag{3}$$

It is clear from the above expression that $+p$ effectively increases and $-p$ decreases the parameter r, thereby changing the dynamics of the logistic map towards increasing complexity with positive pinning and towards decreasing complexity with negative pinning [25].

The diffusively coupled lattice [Eq. (1)] with logistic map defined on each lattice site (logistic CML or LCML) is commonly used as a prototype model for spatially extended systems. It exhibits a wide variety of novel and complex spatiotemporal behaviors including spatiotemporal chaos (STC) for different values of r and coupling strength ϵ [26]. A rough estimate of the strength of pinning required to control the dynamics in LCML can be obtained by linear stability analysis. In the case of uniform pinning, to control spatiotemporal chaos and bring the whole lattice to a fixed point state $(x^* = [(r-1) \pm \sqrt{(r-1)^2 + 4rp}]/2r)$, the strength of pinning required is given by

$$p < \frac{1}{4r}\left[\left(\frac{\epsilon}{1-\epsilon}\right)^2 - (r-1)^2\right]. \tag{4}$$

Thus, for $r=4$, $\epsilon=0.6$, $p<-0.42$. In a similar manner, an estimate of the strength of p required to control the system to any higher periodic states can also be obtained.

The iterates of the logistic map diverge under strong negative pinning [20]. This problem of divergence has been taken care of by applying pinning based on a threshold value ($x_c \leqslant \mathrm{mod}\, p$) of the state variable such that if $x \geqslant x_c$, pinning is not applied. Below we describe the implementation of this control method in the LCML under different conditions.

1. Effect of pinning strength

The salient features of the control approach are summarized in Fig. 1. Figure 1(a) depicts the global dynamic response of the *uniformly pinned* LCML for a range of values of the nonlinear parameter r and pinning strength p. The gray region in this $(r\text{-}p)$ plot shows the parameter values at which the lattice exhibits nonchaotic dynamics as indicated by negative maximum Lyapunov exponent (MLE) s. The regions of different stable dynamical behaviors are marked within the gray region. The black region (positive MLE) indicates chaotic spatiotemporal dynamics. Figure 1(a) clearly shows that, under the influence of negative pinning, the spatiotemporally chaotic dynamics at higher values of r in the LCML are suppressed to lower periodic and fixed-point states and periodic, complex, and chaotic dynamics can be induced in the lattice, at low values of r, by the application

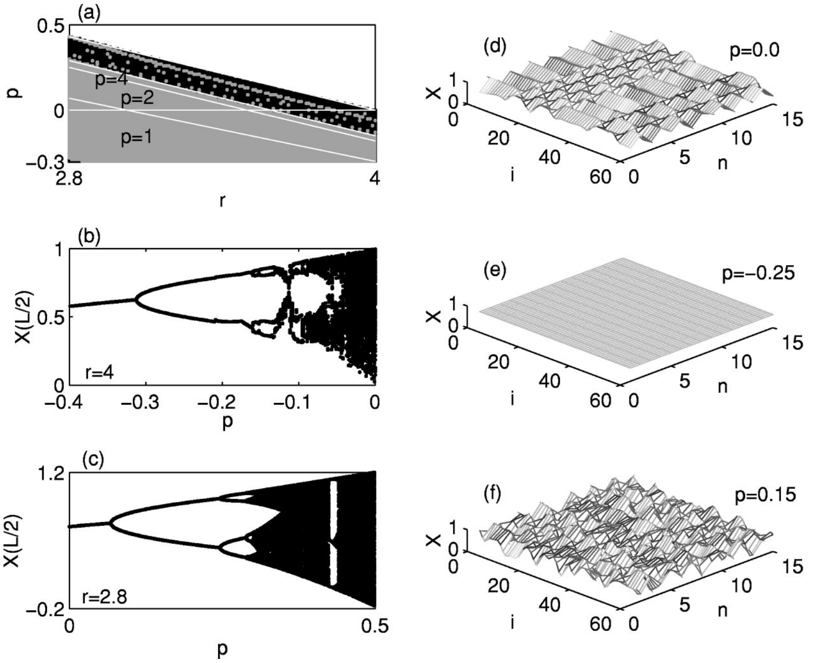

FIG. 1. Global spatiotemporal behavior of LCML under uniform pinning. (a) Dynamics of the LCML in $r-p$ parameter space calculated for r and p in steps of 0.01. The regions marked $p=1$, $p=2$, and $p=4$ in the gray area ($-ve$ MLE) exhibit equilibrium, period-2, and period-4 behavior, and the dark region ($+ve$ MLE) exhibits chaotic dynamics. Bifurcation diagram of site $i=30$ in the CML for (b) $r=4$ as a function of negative p; and (c) $r=2.8$ as a function of positive p. In (b) and (c), $\epsilon=0.6$ and data for 100 consecutive time steps are superimposed after eliminating 1000 time steps. Space-time-amplitude plots of the CML ($r=3.6,\epsilon=0.3$), (d) $p=0$, (e) $p=-0.25$, and (f) $p=0.15$. Initial conditions are randomly chosen from (0,1) for $i=1,2,\ldots,L$.

of positive pinning. Thus it is possible to target any desired spatiotemporal dynamical states in this CML by appropriately choosing the strength and sign of pinning.

The typical local response of a lattice site, to opposing pinning strengths, is plotted in the bifurcation diagrams in Figs. 1(b) and 1(c). In Fig. 1(b) the initial chaotic dynamics of a chosen site exhibits period reversals leading to periodic and fixed-point dynamics under increasing amounts of constant *negative* pinning strength. Conversely, a clear period-doubling behavior to chaos, with increasing strength of *positive p* is observed in a LCML that is initially in a spatiotemporally equilibrium state [Fig. 1(c)]. Similar qualitative behavior is observed at other lattice sites also. Analysis of these local bifurcation plots help in determining the strength and sign of the pinning required for attaining the desired state. The space-time-amplitude plots of a weakly chaotic lattice [Fig. 1(d)] also show complete suppression of chaos under uniform negative pinning [Fig. 1(e)], and enhancement of chaoticity under positive pinning [Fig. 1(f)]. Thus uniform negative and positive pinning can be used to globally control the dynamics of the LCML to desired spatiotemporal behavior.

2. Effect of pinning density

In many real situations it is not possible to have control probes over the entire spatial domain. A useful algorithm should then have the ability to exert control even when applied in lesser density. The effect of spatially *nonuniform pinning* on the spatiotemporal dynamics of the LCML is shown in Fig. 2.

The periodic and weakly chaotic lattices can be easily controlled to the stable state by applying negative pinning at fairly low density (e.g., at every tenth site for a weakly chaotic lattice). But strongly chaotic lattices require a denser distribution of pinning. In Fig. 2(a), the left panel showing a highly turbulent LCML is controlled to temporally equilibrium but spatially periodic dynamics ($s=-0.229$) by pinning every alternate site (central panel). Here the pinned and unpinned sites assume fixed but different values of x (higher for the unpinned site). The right panel shows the controlled nonchaotic dynamics when every third site is pinned ($s=-0.0008$). Suppression of STC is not possible for $i_p>3$ in this highly chaotic lattice even with higher pinning strength. Most real systems do not operate in such highly chaotic state and thus it is possible to control them using this method with low pinning strength applied sparsely over the lattice.

The effect of pinning density in inducing/enhancing chaos is shown for two different values of r in Figs. 2(b) and 2(c). The left panel of Fig. 2(b) shows a stable CML, which, on pinning the alternate sites with positive p, exhibits chaotic dynamics (right panel, $s=0.179$). Similarly, a LCML [left panel of Fig. 2(c)] exhibiting weak, two-band chaos ($s=0.098$) clearly becomes more chaotic ($s=0.409$)—spanning larger area in phase space—when pinned at every alternate site (right panel). We observed that much smaller perturbation was required to enhance chaos in a weakly chaotic lattice in comparison to inducing chaos in a stable lattice. Also it was not possible to destabilize a LCML at equilibrium by pinning fewer than alternate sites.

3. Effect of coupling strength

The coupling strength ϵ is an important parameter whose variation alone can give rise to a wide range of complex spatiotemporal patterns in the system [26]. Thus, for our method to be successful in effecting control over the spatiotemporal system, it would be advantageous to have knowledge of the optimal range of ϵ, especially when the pinning density is low. In Fig. 3 we show the role of ϵ in suppressing fully developed STC [Figs. 3(a)–3(c)], and inducing chaos in a stable lattice [Figs. 3(d)–3(f)] in an alternately pinned LCML, as ϵ is varied from zero (no coupling) to 1 (strongly coupled).

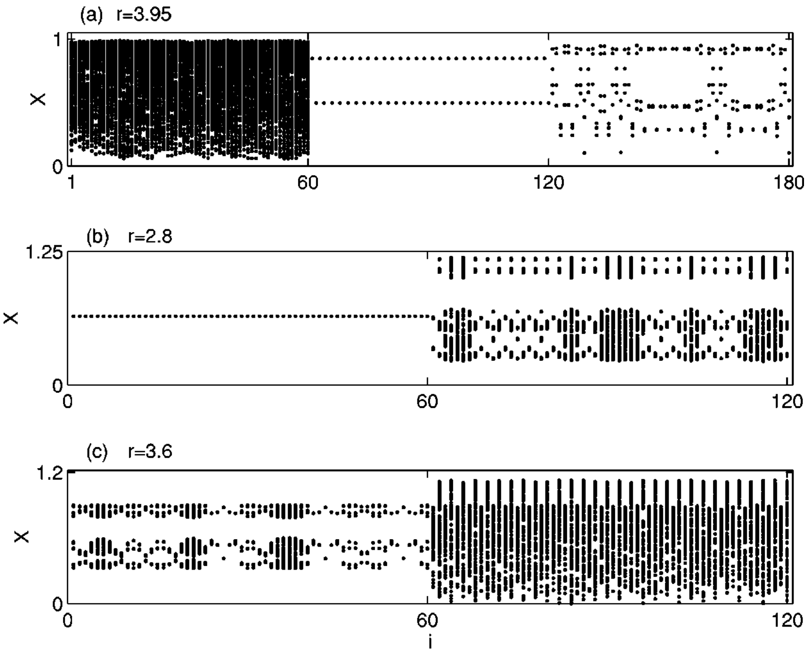

FIG. 2. Space-amplitude plots for LCML under nonuniform pinning. (a) Left, fully developed STC for $r=3.9$; center, temporally stable and spatially periodic dynamics at $p=-0.16$ for $i_p=2$; right, periodic dynamics with $p=-0.4$ for $i_p=3$. Here $\epsilon=0.7$ and $x_c=0.1$. (b) Left, stable LCML with $r=2.8$; right, induced chaotic dynamics for $p=0.5$. (c) Left, weakly chaotic lattice for $r=3.6$; right, enhanced chaos with $p=0.24$. For both (b) and (c) $i_p=2$ and $\epsilon=0.3$.

The bifurcation diagrams in Figs. 3(a) and 3(b) show the temporal behavior of an unpinned (in "gray") site and a neighboring negatively pinned (in "black") site, respectively, in a spatiotemporally chaotic LCML. In absence of any interaction ($\epsilon=0$), the unpinned site shows STC and the neighboring pinned site is controlled to equilibrium state. The neighboring sites interact to influence their respective dynamics as ϵ is increased. At both low and high ϵ, the highly chaotic dynamics in the unpinned site offsets the effect of pinning and the CML is not controllable by this strength of pinning. But at intermediate values of ϵ, a clear period-halving behavior is observed at both the unpinned [Fig. 3(a)] and the pinned [Fig. 3(b)] sites. The typical spatial behavior of this CML, obtained by superimposing data for all the lattice sites at a given time, is shown in Fig. 3(c). This also shows that suppression of chaos is observed only at intermediate coupling strengths for the chosen values of pinning strength and pinning density. It may be recalled that control can always be achieved in a CML by pinning all the sites irrespective of coupling strength.

In order to find the range of ϵ for inducing chaos, we consider a stable LCML whose alternate sites are pinned with positive p [Figs. 3(d)–3(f)]. As in Figs. 3(a)–3(c), in this case also the stable dynamics exerts its influence maxi-

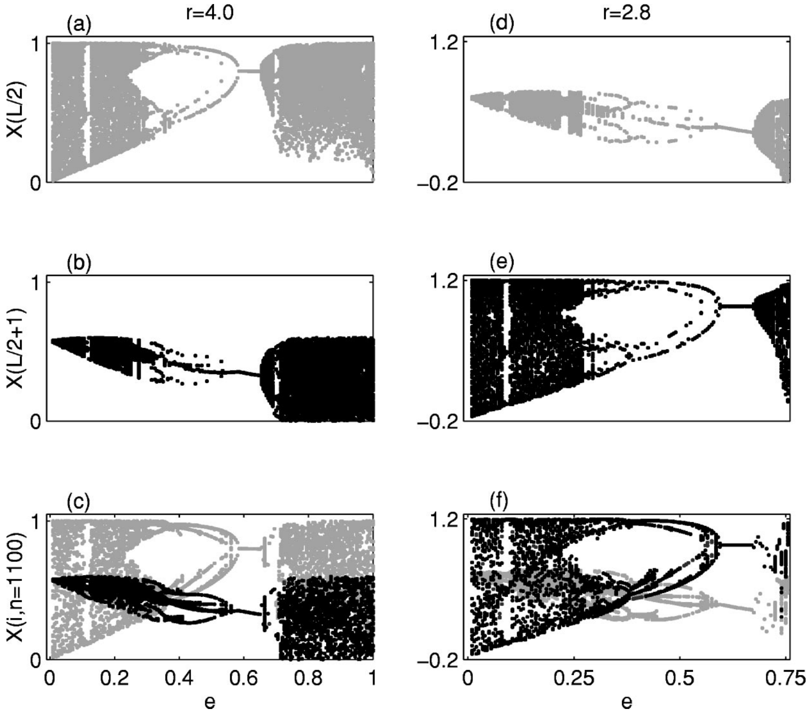

FIG. 3. Effect of coupling strength in a nonuniformly pinned ($i_p=2$) LCML: Suppressing fully developed STC ($r=4$, $p=-0.4$, and $x_c=0.4$) in (a)–(c), and inducing chaos in a stable CML ($r=2.8$, $p=0.5$) in (d)–(f). Temporal behavior of an unpinned site [in "gray" in (a) and (d)], and a pinned site [in "black" in (b) and (e)] from the central region of the CML. (c) and (f) Spatial behavior of the lattices obtained by superimposing data for all the lattice sites at time $n=1100$.

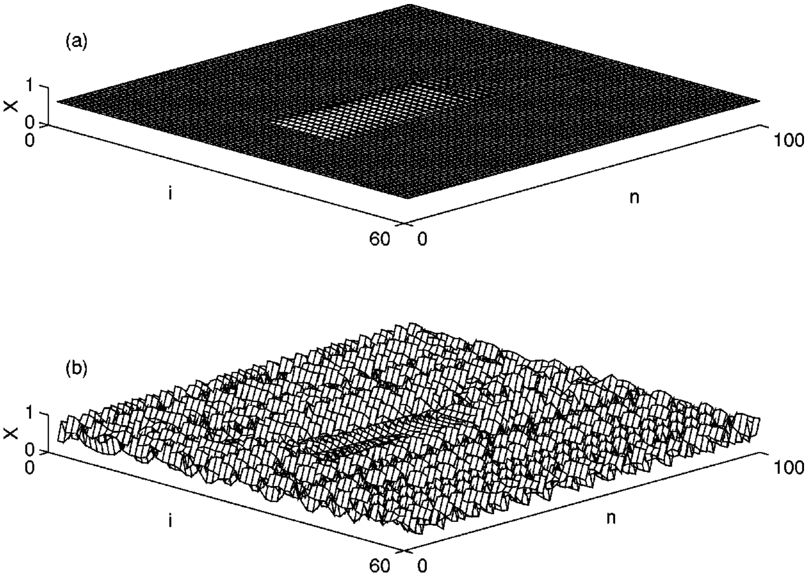

FIG. 4. Space-time-amplitude plots for local control in a LCML ($\epsilon = 0.6$): (a) In a stable CML ($r = 2.8$), r is changed to 3.0 in the central ten sites resulting in periodic oscillations. The altered dynamics is suppressed with $p = -0.04$ applied locally to those sites. (b) In a spatiotemporally chaotic CML ($r = 3.7$), the central ten sites undergo a parameter change ($r = 3.2$) resulting in local reduction in complexity. Application of local positive pinning ($p = 0.21$) restores the chaotic dynamics in the lattice.

mally on moderate values of ϵ. It is clear from the Figs. 3(d)–3(f) that it is possible to induce chaos in a stable LCML only at low and high coupling strengths. In contrast, enhancing the complexity is indeed easier to accomplish in a periodic or weakly chaotic lattice for a larger range of ϵ. The strength of pinning also affects the desired range of ϵ, and stronger pinning strengths allow control over an extended range of coupling strengths.

4. Effect of local pinning

(i) Local control. Global control may not always be the aim of many applications. For example, a diseased state can induce spatially localized changes in biological tissues (e.g., ectopic node in heart or epileptic focus), which then affect their normal functional dynamics [27]. The therapeutic measures involve suppressing such ectopic activities locally. In Fig. 4 we show that our method is quite suitable for restoring spatially localized changes in dynamics to the original state leaving the rest of the system undisturbed.

As shown in Fig. 4(a), localized alteration in the dynamics of a stable lattice was induced by increasing the parameter r slightly in the central ten sites. This introduced small oscillations in that region. On applying a small negative pinning to these sites, complete suppression of the local oscillatory behavior was observed and equilibrium dynamics restored in the whole lattice again. In Fig. 4(b), we consider a chaotic lattice whose central ten sites started exhibiting periodic oscillations due to reduction in r. Application of local positive pinning to these sites allowed the entire lattice to restore its original chaotic behavior. The effect of pinning is quite firmly localized when ϵ is not very high. Thus it is clear that this approach can also be used to induce desired local alterations in spatially extended systems.

(ii) Global control in lattice with nonlocal coupling. Many large spatiotemporal systems possess both local and nonlocal interactions among its different parts, and pinning the entire system to achieve global control is practically impossible. A typical example is a metapopulation in ecology where multiple subpopulations interact through migration. We now show that, using our approach, we can effectively achieve global control over such multiply coupled systems by localized pinning—a property that can have important applications in both real and engineered systems. In Table I we summarize the results of a large number of trials for global control of a spatiotemporally chaotic LCML (having increasing number of random, nonlocal, diffusive links) by randomly pinning a small block of sites. It shows that locally pinning a block of 15 sites, chosen randomly on the lattice, can confer global control in more than 85% cases in lattices with few (about 200) nonlocal connections. Larger pinning region and/or more nonlocal connections are needed for controlling strongly chaotic systems. Similar results are obtained for inducing/enhancing chaos in the lattice, though stronger positive p or larger pinning region is required for lattices with more nonlocal connections as they tend to stabilize the dynamics.

All the results described above for the one-dimensional lattice can be generalized to higher spatial dimensions with appropriate changes in pinning strengths and pinning densities.

B. Case II: Controlling the host-parasitoid CML

The "pinning" approach is equally effective for higher dimensional dynamical systems. Here we show its imple-

TABLE I. Global control by local pinning in LCML with nonlocal coupling: Along with their nearest neighbors, m randomly chosen sites in the lattice ($L = 60$) are diffusively coupled to another n randomly chosen sites in a spatiotemporally chaotic LCML ($r = 3.6$, $\epsilon = 0.2$). Pinning ($p = -0.2$) is applied to a block of 15 sites spanning the whole lattice for each (m and n). The MLE (s) is calculated before (to ensure +ve MLE) and after pinning. Results are averaged over 25 random trials for each (m and n).

m	n	% of −ve MLE
25	5	29
40	5	86
50	5	98

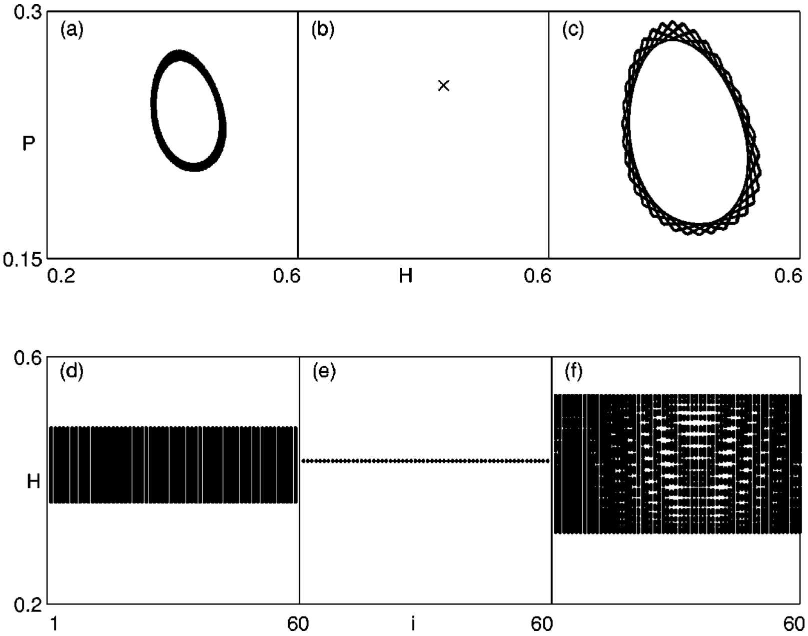

FIG. 5. Control in host-parasitoid CML ($\epsilon=0.2$) on pinning the host: phase plane plots for H and P in a site for (a) $p=0$, (b) 0.03, and (c) $p=-0.02$. The corresponding lattice dynamics in space amplitude plots showing that the unpinned quasiperiodic dynamics (d) is suppressed by positive pinning (e), and enhanced with negative pinning (f) of the host.

mentation for the host-parasitoid (HP) system [28] in ecology, which is described by two coupled discrete equations

$$H(n+1)\equiv f_1=rH(n)[1-H(n)]\exp[-\beta P(n)]$$

$$P(n+1)\equiv f_2=H(n)[1-\exp\{-\beta P(n)\}], \qquad (5)$$

where $H(n)$ and $P(n)$ are the densities of the host and parasite populations at the nth generation. The density-dependent logistic growth of the host population is modulated by parasitism ($\exp[-\beta P(n)]$), and the parasites grow only by infecting the hosts. The parameters r and β represent the intrinsic growth rate of the host and the searching efficiency of the parasitoid.

We consider a model metapopulation of the host-parasitoid system on a one-dimensional lattice where both the hosts and parasitoids diffuse to the nearest neighboring sites. This system exhibits a variety of complex spatiotemporal patterns [28]. Here we present only the results of implementing our control method for the case of uniform pinning to the host variable. The long-term dynamics of the host and parasite at a chosen lattice site in the (HP) phase plane is shown in Figs. 5(a)–(c) and the spatiotemporal dynamics of the host in the lattice is depicted in the space-amplitude plot in Figs. 5(d)–(f). Both the host and the parasite are known to exhibit quasiperiodic dynamics individually and collectively for the parameter values $r=4$ and $\beta=3.5$ as shown in Fig. 5(a) and 5(d). Figures 5(b) and 5(e) show that a small positive pinning (immigration/addition) to the host population can lead to suppression of the complex dynamics at all lattice sites to a fixed-point state. Conversely,

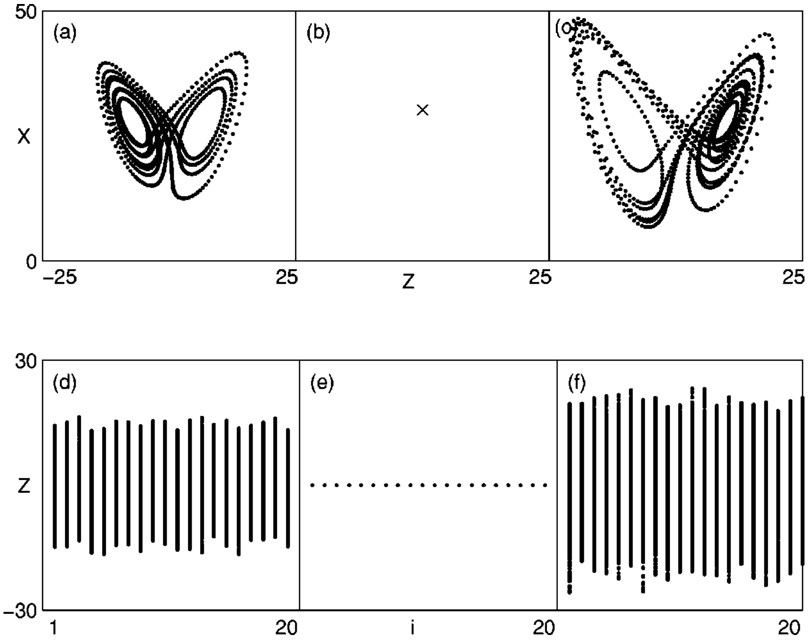

FIG. 6. Control in the spatially extended Lorenz system ($\epsilon=0.3$). Phase plane plots for X and Z in a site for (a) $p=0$, (b) 0.5, and (c) $p=-0.5$. The corresponding lattice dynamics showing the unpinned chaotic dynamics (d) being suppressed by positive pinning (e), and enhanced with negative pinning (f).

Figs. 5(c) and 5(f) show that a small negative pinning (harvesting/removal) of the host enhances the complexity in dynamics, with both the host and parasitoid exhibiting complex and higher amplitude oscillations. One can implement this control method by pinning the parasitoid population also.

C. Case III: Controlling the Lorenz system

As a prototype for a higher-dimensional continuous system exhibiting chaotic dynamics, we choose the Lorenz system [29] described by three coupled differential equations

$$\dot{x} \equiv f_1 = \sigma(y-x),$$

$$\dot{y} \equiv f_2 = -xz + rx - y,$$

$$\dot{z} \equiv f_3 = xy - bz. \tag{6}$$

This system shows low-dimensional chaos for the parameter values $r=28$, $b=2.67$, and $\sigma=10$. We use this dynamical system on a spatially discrete one-dimensional lattice with all three variables diffusing to the nearest neighbors with the same strength. This spatially extended system exhibits spatiotemporally chaotic dynamics for the same parameter values as is shown in the x-z phase plane plot of a representative site [Fig. 6(a)], and the space-amplitude plot of the whole lattice for the variable z [Fig. 6(d)]. Figures 6(b) and 6(e) show that, with a small, uniform, positive pinning to f_3, the chaos in Lorenz system is completely suppressed locally and the lattice also exhibits equilibrium dynamics. And, a small negative pinning enhances the amplitude of the chaotic oscillations both locally [Fig. 6(c)] and globally [Fig. 6(f)]. Thus we show that this method can effectively be applied to spatially extended systems with local continuous dynamical systems.

D. Case IV: Simulation of an experimental result

In experimental situations "pinning" implies perturbing the variable/signal that is being measured. Here we refer to an experimental study [1] that shows the effect of polarity of external electric field on the neuronal electrical activity in hippocampal slices of rats. In this experiment, small external electric fields of opposing polarity were used to modulate the spontaneous burst activity in the pyramidal cell layer in the CA1 and CA3 region of the rat hippocampus. The relationship between both the electric field polarity and the magnitude on the CA3 burst frequency is reproduced in Fig. 7(a). The experiment showed that the burst frequency alternately increased and decreased as a direct result of changes in field polarity—negative field amplitudes accelerating and positive amplitudes suppressing the burst rate.

At a microscopic scale the neuronal tissue can be represented as a discrete network of individual cells with local electrical and chemical processes coupled through gap junctions. We use the logistic map as a formal description of the local complex dynamics of bursting neurons and model the system at base line with a weakly chaotic LCML. We simulate the results shown in Fig. 7(a) by pinning this LCML for different strengths of opposing signs [Fig. 7(b)]. The upper panel in Fig. 7(b) shows the strength of pinning and the lower panel gives the average amplitude of oscillations in the lattice sites. It is indeed obvious that positive pinning clearly induces higher amplitude oscillations mimicking the higher neuronal activity, and negative pinning suppresses the background activity to differing extents depending on the pinning strength.

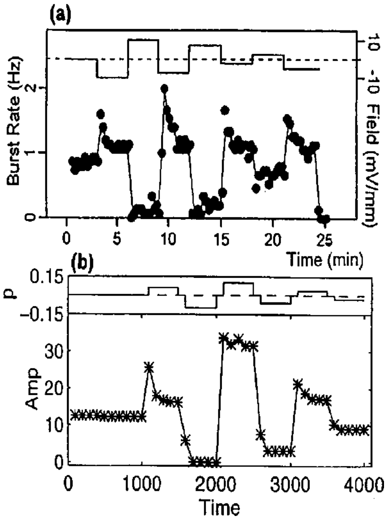

FIG. 7. (a) Effect of polarity of external electric field on bursting neurons (reproduced from Ref. [1] with permission). Burst rate (connected dots) from transverse CA3 under the influence of external electric field (solid line in upper trace) aligned parallel to the dendritic-somatic axis of the pyramidal cells. Burst rate is determined from number of bursts occurring within nonoverlapping 15-s time windows upon switching the polarity of the field. At base line, without applied field, pyramidal cells synchronously discharge at ~1 Hz. (b) Simulation of [1] with a weakly chaotic LCML ($r=3.6, \epsilon=0.3$). The amplitude (Amp) is determined by taking the difference $x_{max}(i) - x_{min}(i)$ and averaging for all sites for 100 consecutive time steps after eliminating the transients. On switching from negative to positive pinning (p in upper trace), the amplitude changes from a large value to a lower value, similar to the bursting neurons. Here, switching from positive to negative pinning is not continuous and appropriate pinning is applied after eliminating transients (500 time steps) for the CML with identical random initial conditions for each case.

IV. CONCLUSION

Most dynamical systems manifest parameter regimes where bifurcation between dynamical states can generate altered dynamics that can be associated with the onset of pathological behavior. Both suppression and preservation of chaotic or complex dynamics are thus important in restoring normalcy. By using our simple approach, we have shown that it is possible to target the spatiotemporal dynamics, in a wide variety of systems under many realistic conditions, to any desired stable fixed-point/periodic or chaotic states by simply applying constant pinning signal of opposite signs in the spatial domain.

In situations where a tractable mathematical model for the system is available, linear stability analysis can be used to obtain a rough estimate of the strength and sign of pinning required for control, as is shown in Eqs. (3) and (4). Alternatively, a few short test experiments on the local functional form can help determine the sign and the range of pinning strength required to achieve the desired performance. Similarly, in the case of experimental systems, one can carry out a few trial experiments by applying external perturbation, of opposing sign and differing strengths, to the signal being measured to get an indication of the efficacy of the control method. The experimental result [1] elaborated above is a representative example of this method as it involves application of external electic field to suppress and enhance the electrical bursting of the neuronal tissue. Similar experimental studies exist where alteration in dynamics is effected simply by perturbing the variable/signal that is being measured directly or through controllers [18,30]. This indicates that this method of control is quite effective and applicable to many fields of science and engineering.

The pinning effectively reduces/increases the nonlinearity/parameter causing the dynamics to move towards stable or unstable manifold in single maps and CML [20,24]. We believe that pinning has a similar effect on other dynamical systems. Persistence of pinning is required in this method which makes the controlled state robust against perturbations. This is a common feature in therapeutic measures of controlling pathologies, where regular application of drugs or the presence of *in situ* pacemakers are required till the diseased part is physically altered or removed. An important aspect of this approach is that it controls the dynamics both locally and globally (for systems with nonlocal couplings) through the application of spatially localized pinning. Such properties allow differential control, i.e., targeting different dynamics in different regions in space and time, and can be very useful in large, heterogeneous, multiply connected systems (e.g., metapopulation in ecology or large arrays of coupled oscillators) for modulating foci of activity, epidemic centers, waves of disturbances or to desynchronize a spatially extended system. Thus, this simple and general approach can be used as a powerful tool to target both stable and complex spatiotemporal dynamics at different spatial and temporal scales allowing possibilities for wide-ranging applications.

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